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$$\text{or } \frac{BG}{GC} = \frac{AB}{AE} \dots (1).$$

Hence, if BG be a multiple of GC then AB will be a multiple of AE . In that case divide BG by GC and AB by AE . Now cut along the line $A1$, 2 3, 4 5 to G and then bring 2 3, into coincidence with $A1$, and GC into coincidence with 4 5. Then the given rectangle has been transformed into a square. This problem is impossible if the ratios in equation (1) are not integers. In other words, if $AB=a$, and $BC=b$, $EB=\sqrt[3]{ab}$, $EA=\sqrt[3]{ab}-a$, $GC=b-\sqrt[3]{ab}$, the equation (1) becomes

$$\frac{\sqrt[3]{ab}}{b-\sqrt[3]{ab}} = \frac{a}{\sqrt[3]{ab}-a} = \text{an integer} \dots (2),$$

and the desired section cannot be performed unless (2) be true.

$$\text{Assume } \frac{a}{\sqrt[3]{ab}-a} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}-\sqrt[3]{a}} = i, \text{ an integer.}$$

Then $\sqrt[3]{b} = \sqrt[3]{a} (1 + \frac{1}{i})$, which is satisfied when $i=1$ only.

345. Proposed by LLOYD HOLSINGER, Bradley Polytechnic Institute, Peoria, Ill.

If a variable polygon move in such a way that its n sides turn severally round n fixed points $O_1, O_2, \dots, O_n$ while $n-1$ of its vertices slide, respectively, along $n-1$ fixed straight lines $v_1, v_2, \dots, v_{n-1}$, then the last vertex will describe a conic; and the locus of the point of intersection of any pair of non-adjacent sides will also be a conic. Cremona's *Projective Geometry*.

No solution of this problem has been received.

346. Proposed by G. I. HOPKINS, M. A., Professor of Mathematics and Astronomy, High School, Manchester, N. H.

Prove the theorem for finding the lateral area of a frustum of a cone without the use of the theory of limits.

I. Solution by the PROPOSER.

The theorem relating to the lateral area of the frustum of a cone is given in most text books on Solid Geometry, and proved by the theorem of limits. The accompanying proof does not use limits. The lateral area of the frustum of a cone is equal to its slant height multiplied by one half the sum of the circumferences of its bases.

Let $BADE$ be the frustum of the cone OAD . Designate the radii of its bases by R and r , their circumferences by C and c . Also designate the lateral areas of the two cones by L and l , and their slant heights by S and s .

Then $S-s$ represents the slant height of the frustum, and $L-l$ its lateral area.

$$L = \frac{CS}{2} \text{ and } l = \frac{cs}{2}. \quad \therefore L - l = \frac{CS - cs}{2}.$$

$$\frac{S}{s} = \frac{R}{r} = \frac{C}{c}. \quad \therefore Sc = Cs.$$

$$\therefore Sc - Cs = 0. \quad CS - cs = CS - cs.$$

$$\therefore CS + Sc - Cs - cs = CS - cs. \quad \therefore (C + c)(S - s) = CS - cs.$$

$$\therefore L - l = \frac{(S - s)(C + c)}{2}. \quad \text{Q. E. D.}$$

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Complete the frustum to a full cone, and by cutting along an element spread out the surface into a plane, which can be done with all developable surfaces. The circular arc AB is equal to the circumference of one of the base circles of the frustum $= 2\pi R$, and arc $CD = 2\pi r$.

$AC = BD =$ slant height l of frustum.

From $OA : OC = AB : CD = R : r$, and $OA - OC = l$, we get

$$OA = \frac{Rl}{R - r}, \quad OC = \frac{rl}{R - r}.$$

$$\begin{aligned} \text{Area of } ABCD &= \frac{1}{2}AB \cdot OA - \frac{1}{2}CD \cdot OC = \frac{1}{2} \cdot 2\pi R \cdot \frac{Rl}{R - r} - \frac{1}{2} \cdot 2\pi r \cdot \frac{rl}{R - r} = \\ &= \pi l \cdot \frac{R^2 - r^2}{R - r} = \pi l(R + r). \end{aligned}$$

CALCULUS.

277. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

Find $\frac{d^2x}{ds^2}$ and $\frac{d^2y}{ds^2}$ for $y = c \sinh \frac{x}{c}$.

Solution by J. W. CLAWSON, Collegeville, Pa.

$$y = c \sinh \frac{x}{c}. \quad \therefore \frac{dy}{dx} = \cosh \frac{x}{c}. \quad \text{But } ds^2 = dx^2 + dy^2.$$

$$\therefore \left(\frac{ds}{dx} \right)^2 = 1 + \cosh^2 \frac{x}{c} \quad \text{and} \quad \left(\frac{ds}{dy} \right)^2 = 1 + \frac{1}{\cosh^2 (x/c)}.$$

$$\therefore \frac{dx}{ds} = (1 + \cosh^2 \frac{x}{c})^{-\frac{1}{2}} \quad \text{and} \quad \frac{dy}{ds} = (1 + \cosh^2 \frac{x}{c})^{-\frac{1}{2}} \cdot \cosh \frac{x}{c}.$$

$$\therefore \frac{d^2x}{ds^2} = -\frac{1}{2} (1 + \cosh^2 \frac{x}{c})^{-\frac{3}{2}} \cdot 2 \cosh \frac{x}{c} \sinh \frac{x}{c} \cdot \frac{1}{c} \cdot (1 + \cosh^2 \frac{x}{c})^{-\frac{1}{2}}$$